Heat and Mass Transfer Flow of a Micropolar Fluid Under Convective Boundary Condition from a Horizontal Cylinder with Temperature Dependent Viscosity and Chemical Reaction

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The effects of variable viscosity and chemical reaction on heat and mass transfer by free convection boundary-layer flow of a micropolar fluid past a horizontal circular cylinder with thermal convective surface is analyzed. The governing partial differential equations are transformed into a set of non-similar equations which are solved numerically by an implicit finite-difference method. Comparisons with previously published works are performed and excellent agreement is obtained. A parametric study of the physical parameters is conducted and a representative set of numerical results for the velocity, angular velocity, temperature, and concentration fields as well as the local skin-friction coefficient, wall couple stress, local Nusselt and Sherwood numbers is illustrated graphically to show interesting features of the solutions.

Keywords: Free Convection, Micropolar Fluids, Variable Viscosity, Chemical Reaction, Boundary Layer Flow.

1. INTRODUCTION
Coupled heat and mass transfer by natural convection in a fluid saturated porous medium has many important applications in geothermal and geophysical engineering such as the extraction of geothermal energy, the migration of moisture in fibrous insulation, under ground disposal of nuclear waste, and the spreading of chemical pollutants in saturated soil. In addition, micropolar convection flows in porous media have received less attention despite important applications in emulsion filtration, polymer gel dynamics in packed beds, petroleum and lubrication flows in porous wafers. The theory of micropolar fluid was first proposed by Eringen.\textsuperscript{1} Extensive review of the theory and applications can be found in the review article by Ariman et al.\textsuperscript{2} and Cowin.\textsuperscript{3} Patil and Hiremath\textsuperscript{4} have studied the effects of couple stresses on the flow of micropolar fluids through a porous medium. Raptis and Takhar\textsuperscript{5} have examined the problem of Steady flow of a polar fluid through a porous medium. Ogulu\textsuperscript{6} discussed the problem of the micropolar flow past a vertical porous plate in the presence of couple stresses. EL-Kabeir and Gorla\textsuperscript{7} have studied the MHD effect on natural convection in a micropolar fluid at a three-dimensional stagnation point in a porous medium. Cheng\textsuperscript{8} analyzed the the natural convection flow of micropolar fluid near a vertical truncated cone. Cheng-Long Chang\textsuperscript{9} considered numerical simulation for natural convection of micropolar fluids flow along slender hollow circular cylinder. Pal\textsuperscript{10} studied the problem of heat and mass transfer in MHD non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous media. EL-Kabeir et al.\textsuperscript{11} have considered the heat transfer in a micropolar fluid flow past a porous continuous moving surface. Rashidi et al.\textsuperscript{12} have obtained the analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation.

On other hand, coupled combined heat and mass transfer problems with chemical reactions are of importance in many processes and therefore have received a considerable amount of attention in recent years. A reaction is said to be the first-order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glassware. Das et al.\textsuperscript{13} studied the effect of first-order chemical reaction on the
Heat and mass transfer flow past an impulsively started infinite vertical plate. Kandasamy et al.\textsuperscript{14} have investigated the heat and mass transfer effects along wedge with heat source and concentration in the presence of the chemical reaction of the first-order. The problem of heat and mass transfer by MHD non-Darcian flow of a micropolar fluid over a stretching surface embedded in a porous media is analyzed by Pal and Chatterjee.\textsuperscript{15} EL-Kabeir and Modather\textsuperscript{16} studied the effect of chemical reaction on heat and mass transfer flow over a vertical isothermal cone surface in micropolar fluids. Patil and Kulkarni\textsuperscript{17} have studied the effect of chemical reaction by free convection flow of a micropolar fluid through a porous medium. Damseh et al.\textsuperscript{18} have examined the combined effects of heat and chemical reaction on micropolar fluid flows over a uniformly stretched porous surface. An analytical study on heat and mass transfer oscillatory flow of micropolar fluid over a vertical permeable surface in micropolar fluids. Rashad et al.\textsuperscript{24} have also analyzed the effect of chemical reaction on coupled heat and mass transfer by natural convection flow of a micropolar fluid saturated porous medium. Rashad et al.\textsuperscript{25} have investigated the effects of radiation and chemical reaction on heat and mass transfer by natural convection in a micropolar fluid saturated porous medium. Rashad et al.\textsuperscript{26} have also analyzed the effect of chemical reaction on, mixed convection flow of a micropolar fluid over a continuously moving vertical surface immersed in a thermally and solutally stratified medium.

Therefore, the aim of the present paper is to study the effect of chemical reaction on coupled heat and mass transfer by natural convection flow of a micropolar fluid past a horizontal cylinder embedded in a porous medium considering Brinkman-extended Darcy model with variable viscosity under convective boundary condition. The order of chemical reaction in this work is taken as a first-order reaction. The full governing boundary layer equations are first transformed into a system of dimensionless equations via the dimensionless variables, and then into non-similar partial differential equations before they are solved numerically by an implicit finite-difference. Numerical results are presented for velocity, angular velocity, temperature and concentration profiles within the boundary layer for different parameters entering into the analysis. Also the effects of the pertinent parameters on the local skin friction coefficient, wall couple stress, local Nusselt and Sherwood numbers are also discussed.

### 2. GOVERNING EQUATIONS

Consider the steady, two-dimensional, laminar natural convection heat and mass transfer flow of micropolar fluid past a horizontal cylinder radius $a$ saturated porous medium with variable viscosity in the presence of chemical reaction.

![Fig. 1. Problem schematics and coordinate system.](image)

![Fig. 2. Effect of $\Delta$ on velocity profiles.](image)

**Table I.** Comparison of $-\theta' (0)$ for different values of $x$ at $Pr = 1.0$ for Newtonian fluid ($\Delta = 0.0$) at $\lambda = e = 0$ and $Bi, Da \to \infty$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Merkin\textsuperscript{13}</th>
<th>Yih\textsuperscript{26}</th>
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Fig. 3. Effect of \( \Delta \) on temperature profiles.

effect. Figure 1 shows the flow model and physical coordinate system, where \( x \)-coordinate is measured along the circumference of the horizontal cylinder of from the lowest point and the \( y \)-coordinate is measured normal to the surface, respectively. It is assumed that the surface of cylinder is heated by convection from a hot fluid at temperature \( T_f \) which provides a heat transfer coefficient \( h_f \) and the temperature of the ambient fluid is \( T_0 \). In addition, the concentration of a certain constituent in the solution varies from \( C_0 \) on the micropolar fluid side of the cylinder surface to \( C_\infty \) sufficiently far from the cylinder surface. A first-order homogeneous chemical reaction is assumed to take place in the flow. The micropolar fluid properties are assumed to be constant except for density variations in the buoyancy force term and the fluid viscosity depends on temperature. Under the Boussinesq and boundary layer approximations, the basic equations for the equations for mass, momentum, angular momentum, energy and concentration can be written as: (see and Merkin\(^{25}\) and Yih\(^{26}\))

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0
\]  

Fig. 4. Effect of \( \Delta \) on concentration profiles.

\[
\rho \left( \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = \frac{\partial}{\partial y} \left( \mu + K \frac{\partial \bar{u}}{\partial y} + K \frac{\partial \bar{N}}{\partial y} \right) + \rho g \left[ \beta_1 (T - T_0) + \beta_2 (C - C_\infty) \right] \sin \frac{\pi}{2} \bar{u} + \frac{\mu + K}{K_{1}} \bar{u} \quad (2)
\]

\[
\bar{u} = \bar{v} = 0, \quad \bar{N} = 0, \quad -\frac{\partial T}{\partial y} = h_f (T_f - T) \quad (6)
\]

\[
\bar{u} \to 0, \quad \bar{N} \to 0, \quad T \to T_0 \quad (6)
\]

\[
C \to C_\infty \quad \text{as} \quad \bar{y} \to \infty
\]

The corresponding boundary conditions for this problem can be written as:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0
\]

Fig. 5. Effect of \( \Delta \) on microrotation profiles.

\[
\bar{u} = \bar{v} = 0, \quad \bar{N} = 0, \quad -\frac{\partial T}{\partial y} = h_f (T_f - T) \quad (6)
\]

\[
\bar{u} \to 0, \quad \bar{N} \to 0, \quad T \to T_0 \quad (6)
\]

\[
C \to C_\infty \quad \text{as} \quad \bar{y} \to \infty
\]

Fig. 6. Effect of \( \Delta \) on the skin-friction coefficient.
where $\vec{u}$ and $\vec{v}$ are the velocity components along $\vec{x}$ and $\vec{y}$ axes, $\vec{N}$ is the component of the microrotation vector normal to the $\vec{x} - \vec{y}$, $T$ is the micropolar fluid temperature, $C$ is the fluid concentration, $\gamma^*$ is the magnitude of the acceleration due to gravity, $\beta_T$ is the coefficient of the thermal expansion; $\beta_C$ is the coefficient of the concentration expansion, $\rho$ is the density, $\mu$ is the viscosity of the fluid, $\alpha$ is the thermal diffusivity of the ambient fluid, $D$ is the mass diffusivity, $h_f$ is the convective heat transfer coefficient, $k$ is the thermal conductivity, $K_1$ is the permeability of the porous medium, $K$ is the vortex viscosity, $\gamma^*$ is the spin-gradient viscosity, $j$ is the microinertia density, $k_f$ is the chemical reaction.

The viscosity of the fluid is assumed to be an inverse linear function of temperature and it can be expressed as following Ling and Dybbs\textsuperscript{27} and Molla et al.\textsuperscript{28}

$$\mu = \frac{\mu_\infty}{1 + b(T - T_\infty)}$$

where $b$ is a constant and $\mu_\infty$ is the viscosity of the ambient fluid. We shall now proceed to transform Eqs. (1)-(5) to a form amenable for numerical solution. The governing Eqs. (1)-(5) and boundary conditions (6) can be made dimensionless by introducing the following dimensionless variables

$$x = \frac{\vec{x}}{a}, \quad y = \frac{\vec{y}}{a} Gr^{1/4}, \quad u = \frac{\rho a}{\mu_\infty} Gr^{-1/4} \vec{u}$$

$$v = \frac{\rho a}{\mu_\infty} Gr^{-1/4} \vec{v}, \quad N = \frac{\rho a^2}{\mu_\infty} Gr^{-3/4} \vec{N}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad Gr = \frac{\rho \beta(T_f - T_\infty) a^3}{\nu^2}, \quad \nu = \frac{\mu_\infty}{\rho}$$

where $Gr$ is the Grashof number, $\nu$ is the reference kinematic viscosity, $x$ and $y$ are dimensionless coordinates, $u$ and $v$ are dimensionless velocities, $\theta$ is the dimensionless temperature, $N$ is the dimensionless microrotation, and $\phi$ is the dimensionless concentration. Following the work of Rees and Pop,\textsuperscript{29} $\gamma^*$ is assumed to be given by

$$\gamma^* = \left( \mu + \frac{K_j}{2} \right) j$$
Substituting Eq. (8) into Eqs. (1)–(5), we obtain the coupled, nonlinear, dimensionless partial differential equations for the regime as follows:

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{(1 + \varepsilon \theta)} \left[ \frac{\partial^2 u}{\partial y^2} - \frac{\varepsilon}{(1 + \varepsilon \theta)^2} \frac{\partial u}{\partial y} \right] + \frac{\Delta \partial N}{\partial y} - Da^{-1} \left( \frac{1}{1 + \varepsilon \theta} + \frac{\Delta}{2} \right) \frac{\partial u}{\partial y}
\]

\[
+ \left( \theta + \Delta \phi \right) \sin x
\]

\[
\left( \frac{1}{1 + \varepsilon \theta} + \frac{\Delta}{2} \right) \frac{\partial^2 N}{\partial y^2} = \frac{\varepsilon}{(1 + \varepsilon \theta)^2} \frac{\partial N}{\partial y} - Da^{-1} \left( \frac{1}{1 + \varepsilon \theta} + \frac{\Delta}{2} \right) \frac{\partial N}{\partial y}
\]

\[
- Da^{-1} \left( 2N + \frac{\partial u}{\partial y} \right) = u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y}
\]

\[
\frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \gamma \phi
\]

Fig. 11. Effects of \( y \) and \( \varepsilon \) on temperature profiles.

With the boundary conditions

\[
u = 0, \quad N = 0, \quad \frac{\partial \theta}{\partial y} = -Bi(1 - \theta)
\]

\[
\phi = 1 \quad \text{at} \quad y = 0
\]

\[
u = 0, \quad N = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{as} \quad y \rightarrow \infty
\]

where \( Da \) is the Darcy number, \( \Lambda \) is the concentration to thermal buoyancy ratio, \( \gamma \) is the vortex viscosity parameter, \( \varepsilon \) is the dimensionless chemical reaction parameter, \( \varepsilon \) is the viscosity parameter, \( Bi \) is the Biot number, \( B \) is the spin gradient, \( Pr \) is the Prandtl number and \( Sc \) is the Schmidt number which are respectively given by:

\[
Da^{-1} = \frac{\alpha^2 Gr^{-1/2}}{K_i}, \quad \Lambda = \frac{(C_w - C_{in})\beta_c}{(T_f - T_{in})\beta_T}, \quad \Delta = \frac{K}{\mu_{\infty}}
\]

\[
\gamma = \frac{k_c a^2}{\nu_{\infty} Gr_{1/2}}, \quad \varepsilon = \gamma(T_w - T_{in}), \quad Bi = \frac{h_l a}{k Gr^{1/2}}
\]

\[
B = \frac{\alpha^2 Gr^{-1/2}}{j}, \quad Pr = \frac{\nu_{\infty}}{\alpha}, \quad Sc = \frac{\nu_{\infty}}{D}
\]
To solve Eqs. (10)–(14) with the boundary conditions (15) we introduce the following variables:

\[
\psi = xf(x, y), \quad N = xg(x, y) \\
\theta = \theta(x, y), \quad \phi = \phi(x, y)
\]

(17)

where \( \psi \) is the non-dimensional stream function defined in the usual way as; \( u = \partial \psi / \partial y, \ v = -\partial \psi / \partial x \). Therefore, the continuity equation is automatically satisfied. Substituting (17) into Eqs. (11)–(14), we get the following transformed equations:

\[
\frac{1}{1 + e\theta + \Delta} \frac{\partial^3 f}{\partial x^3} + \frac{e}{(1 + e\theta)^2} \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x^2} - f' + \epsilon f'' + \Delta g' = 0
\]

(18)

\[
-\frac{1}{\Pr} \frac{\partial \theta}{\partial x} + \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x^2} + f' \frac{\partial f}{\partial x} = x \left( f' \frac{\partial f}{\partial x} - f'' \frac{\partial f}{\partial x} \right)
\]

(19)

\[
\frac{1}{\Sc} \frac{\partial \phi}{\partial x} + \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial x} = x \left( f' \frac{\partial \phi}{\partial x} - \phi' \frac{\partial f}{\partial x} \right)
\]

(20)

\[
\frac{1}{\Sc} \frac{\partial \phi}{\partial x} + \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial x} = x \left( f' \frac{\partial \phi}{\partial x} - \phi' \frac{\partial f}{\partial x} \right)
\]

(21)

The transformed boundary conditions may be written as:

\[
f(x, 0) = f'(x, 0) = g(x, 0) = 0 \quad \theta'(x, 0) = \frac{\partial \theta(x, 0)}{\partial x} = \frac{\partial f}{\partial x} = 1 \quad f'(x, \infty) = g(x, \infty) = \theta(x, \infty) = \phi(x, \infty) = 0
\]

(22)

where the primes denote the differentiation with respect to \( y \). In practical applications, the physical qualities of principal interest are the local skin-friction coefficient \( \Cf \), local Nusselt number \( \Nu \), local wall couple stress \( \Mw \),
and the local Sherwood number $Sh$ are important physical properties. These can be defined in dimensionless form below:

$$C_f = \frac{Gr^{3/4}a^{2}}{\mu \nu_{\infty}} \tau_w, \quad \tau_w = \left( \mu + K \right) \left( \frac{\partial \theta}{\partial y} + K \bar{N} \right)_{y=0}$$  \hspace{1cm} (23)

$$M_w = \frac{Gr^{-1/2} \rho a}{\nu_{\infty}^2} \eta_w, \quad \eta_w = \gamma \left( \frac{\partial \bar{N}}{\partial y} \right)_{y=0}$$  \hspace{1cm} (24)

$$Nu = \frac{a Gr^{-1/4}}{k(T_w - T_{\infty})} \eta_w, \quad \eta_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}$$  \hspace{1cm} (25)

$$Sh = \frac{a Gr^{-1/4}}{D(D_w - D_{\infty})} q_w, \quad q_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0}$$  \hspace{1cm} (26)

Using the variables in Eq. (8), Eq. (17) and the boundary condition Eq. (22), we get the following expressions for $C_f$, $Nu$, $M_w$, and $Sh$

$$C_f = x \left[ \frac{1}{1 + \varepsilon \theta(x, 0) + \Delta} \right] f'(x, 0)$$  \hspace{1cm} (27)

$$M_w = \frac{x}{B} \left( \frac{1}{1 + \varepsilon \theta(x, 0) + \Delta} \right) g'(x, 0)$$  \hspace{1cm} (28)

$$Nu = -\theta'(x, 0)$$  \hspace{1cm} (29)

$$Sh = -\bar{N}'(x, 0)$$  \hspace{1cm} (30)

3. RESULTS AND DISCUSSION

The system of Eqs. (18)–(21) subject to boundary conditions (22) have been solved numerically by means of an efficient, iterative, tri-diagonal implicit finite-difference method which was introduced by Blottner. The numerical solutions start from the lower stagnation point of the cylinder, $x = 0$ to the rear stagnation point, $x = \pi$. In order to access the accuracy of the numerical results, a comparison with previously published works reported by Merkin and Yih for the case of Newtonian fluid ($\Delta = 0$) with constant viscosity ($\varepsilon = 0$) over an isothermal circular cylinder ($Bi \to \infty$) in the absence of the buoyancy force and chemical reaction effects ($\Lambda = \gamma = 0$) was performed with various values of $x$ at $Pr = 1.0$. This comparison is presented
in Table I. It is obvious from this table that excellent agreement between the results exists.

In this section, a detailed parametric study has been performed for the effects of the micropolar vortex viscosity parameter $\Delta$, Darcy number $Da$, viscosity parameter $\varepsilon$, Biot number $Bi$, and dimensionless chemical reaction parameter $\gamma$ on the velocity, microrotation, temperature, and concentration profiles as well as the local skin-friction coefficient, wall couple stress $M_w$, Nusselt number $Nu$ and the Sherwood number $Sh$ is presented graphically in Figures 2 through 17. All data are provided in the legends of these figures correspond to a micropolar fluid having a high vortex viscosity ($\Delta = 2$) i.e., is strongly non-Newtonian (unless otherwise indicated) with $B = 1.0$. Since $\Lambda = 5$ in these figures, the thermal and species buoyancy forces are of the same order of magnitude and assist each other. Pr is set as 7.0 corresponding to water and $Sc = 0.6$ implies a dominance of momentum diffusivity over species diffusivity.

Figures 2–9 show the effects of the vortex viscosity parameter $\Delta$ on the velocity, microrotation, temperature, and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number. It is observed that an increase in the vortex viscosity parameter leads to increase the value of temperature, concentration and skin-friction coefficient while the values of velocity, microrotation, wall couple stress, Nusselt number and the Sherwood number decrease. These figures show that velocity, microrotation, wall couple stress, Nusselt number and the Sherwood number are higher in the case of Newtonian fluids ($\Delta = 0$) than the case of micropolar fluids because the resistance of the micropolar fluid is larger than Newtonian fluids as a result of vortex viscosity. An increasing in the value of vortex viscosity parameter produces more resistance of the micropolar fluid which leads to increase the value of temperature, concentration and skin-friction coefficient.

Figures 10–17 depict the influences of the dimensionless chemical reaction parameter $\gamma$ and viscosity parameter $\varepsilon$ on the velocity, microrotation, temperature, and concentration profiles as well as the skin- friction coefficient, wall couple stress, Nusselt number and the Sherwood number. We observe that the temperature profiles increases with an increase in chemical reaction parameter while the velocity, microrotation and concentration profiles decrease, on the other hand the values of the skin-friction coefficient and Nusselt number decrease as the chemical reaction parameter increases and wall couple stress, Sherwood number have the opposite behaviour. We also notice that the velocity, microrotation, wall couple stress, Nusselt number and the Sherwood number increase as viscosity parameter increases while the values of temperature, concentration and the skin-friction coefficient decrease with this increasing.

Figures 18–25 illustrate the velocity, microrotation, temperature, and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number for various values of Darcy number $Da$ and Biot number $Bi$. It is clear from these figures that the velocity, skin-friction coefficient, Nusselt number and the Sherwood number increase as both of Darcy number and Biot number increase while concentration profile...
and wall couple stress decrease with this increase. We also observe that the temperature decreases as Darcy number increases while it increases with the increasing of the value of Biot number. An increasing in values of Darcy number leads to higher porosity porous medium which exerts less resistance to flow, this causes an increasing in the velocity, skin-friction coefficient, Nusselt number and the Sherwood number, on contrast the presence of porous medium (as Da decreases) has the tendency to increase the fluid concentration and temperature. This behavior is a result of in the increment of the fluid restriction due to the decreasing in the porosity of the porous medium.

4. CONCLUSION

In this paper, the effects of chemical reaction and variable viscosity on coupled heat and mass transfer by natural convection flow of a micropolar fluid past a horizontal cylinder saturated porous medium under convective boundary condition. The governing boundary-layer equations were transformed into a non-similar form, and these equations were solved numerically using an implicit finite-difference method. The effects of the Darcy number, Biot number, variable viscosity, chemical reaction, and vortex viscosity parameters on the velocity, microrotation, temperature and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number were shown graphically and discussed. An increase in the vortex viscosity parameter leads to increase the values of temperature, concentration and skin-friction coefficient while the values of velocity, microrotation, wall couple stress, Nusselt number and the Sherwood number decrease. The temperature profiles increase with an increase in chemical reaction parameter while the velocity, microrotation and concentration profiles decrease as it increases, on the other hand the values of the skin-friction coefficient and Nusselt number decrease as the chemical reaction parameter increases and wall couple stress, while the velocity, skin-friction coefficient, Nusselt number and the Sherwood number increase as both of Darcy number and Biot number increase.

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References


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