FORECASTING THE PERFORMANCE OF TADAWUL ALL SHARE INDEX (TASI) USING GEOMETRIC BROWNIAN MOTION AND GEOMETRIC FRACTIONAL BROWNIAN MOTION

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Abstract

It is known that the market indices of Saudi Arabia which is called Tadawul All Share Index (TASI) reflect the performance of economic growth and financial stability of Saudi Arabia. Thus, the forecasting of the performance is quite important. In this empirical study, we forecasted daily index prices of TASI for year 2018. To act this, we depended on two models including geometric Brownian motion and geometric fractional Brownian motion.
(GBM) and geometric fractional Brownian motion (GFBM). Further, the calculation of each model was obtained reliant on three different ways of computing volatility including simple volatility, log volatility and stochastic volatility. Meanwhile, the evaluation of the performance of each model was calculated by using mean absolute percentage error (MAPE). The results revealed that all models have high accuracy with trivial difference. This indicates that all models can be used to forecasting the performance of TASI.

1. Introduction

In 1964, Samuelson [21] introduced a geometric Brownian motion (GBM) which is considered as one of the most significant models in financial mathematics. This model is widely used as the underlying process of a risky market:

\[ dS_t = \mu S_t dt + \sigma S_t dW_t, \]  

(1)

where \( S_t \) represents the stock price process, \( \mu \) mean of return, \( \sigma \) constant volatility and \( W_t \) Brownian motion process.

This model assumed that the volatility (or standard deviation) is constant. This assumption had been criticized and rejected by most empirical studies (such as Bakshi and Tso [7], Aït-Sahalia and Lo [3], Lo [16] and Stein [24]) which led to some market crashes for instance Black-Monday in 1987, the Asian crisis in 1989 and housing bubble and credit crisis 2007-2009. Thus, GBM was later studied under the assumption of stochastic volatility \( \sigma(Y_t) \) such as Scott [22], Hull and White [12], Stein and Stein [23], Heston [11], Comte and Renult [9] and Chronopoulou and Viens [8]:

\[ dS_t = \mu S_t dt + \sigma(Y_t) S_t dW_t, \]  

(2)

where \( Y_t \) represents stochastic process.

Besides, scholars exhibited that the time series data depending on this model revealed the existence of memory (some trend-like behavior). This revelation implied to incorporate the long memory parameter which is
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called *Hurst parameter* (H) into GBM model. This modified model is called *geometric fractional Brownian motion* (GFBM). To simplify the calculations, Misiran et al. [17, 18] and Kukush et al. [13] studied this model under the assumption of constant volatility:

\[
dS_t = \mu S_t dt + \sigma S_t dB_H,
\]

where \( B_H \) represents fractional Brownian motion process with Hurst index \( H > 0.5 \).

However, as we mentioned above, the assumption of constant volatility was rejected by empirical studies. This encouraged Alhagyan et al. [4-6] going ahead to next natural step, i.e., studying GFBM under stochastic volatility assumption:

\[
dS_t = \mu S_t dt + \sigma(Y_t) S_t dB_H.
\]

### 2. Models Under Study

This article compares between the performance of GBM and GFBM models with some volatility formulation available in the literature. GBM models include GBM model with constant volatility computed by simple volatility formula (GBM-S), GBM with constant volatility computed by log volatility formula (GBM-L), and GBM with stochastic volatility computed by a deterministic function \( \sigma(Y_t) = Y_t \) (GBM-STO) where the stochastic process \( Y_t \) obeys fractional Ornstein-Uhlenbeck process with mean-reverting property. Meanwhile, GFBM models include GFBM model with constant volatility computed by simple volatility formula (GFBM-S), GFBM with constant volatility computed by log volatility formula (GFBM-L), and GFBM with stochastic volatility computed by the deterministic functions \( \sigma(Y_t) = Y_t \) (GFBM-STO) where the stochastic process \( Y_t \) obeys fractional Ornstein-Uhlenbeck process with mean-reverting property. Table 1 shows formulations of these volatilities under study.
Table 1. Formulas of volatility

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple volatility ($S$)</td>
<td>$\sigma = \sqrt{\frac{1}{(n-1)\Delta t} \sum_{i=1}^{n} (R_i - \bar{R})^2}$</td>
</tr>
<tr>
<td>Log volatility ($L$)</td>
<td>$\sigma = \sqrt{\frac{1}{(n-1)\Delta t} \sum_{i=1}^{n} (\log(S_i) - \log(S_{i-1}))^2}$</td>
</tr>
<tr>
<td>Stochastic volatility (STO)</td>
<td>$\sigma(Y_t) = Y_t$</td>
</tr>
<tr>
<td></td>
<td>$dY_t = \alpha(m - Y_t) dt + \beta dB_{H_2}(t)$</td>
</tr>
</tbody>
</table>

To evaluate the models, we utilize mean absolute percentage error (MAPE) for forecasting stock indices as per the works of Walsh and Tsou [26], Lam et al. [14] and Abidin and Jaffar [1, 2]. MAPE is a measure of prediction accuracy of a forecasting method. It is the most commonly used measure of assessing forecasts in organizations (Tofallis [25]). According to Abidin and Jaffar [1, 2], the formula of MAPE is given by:

$$\text{MAPE} = \frac{\sum_{i=1}^{n} |Y_i - F_i|}{Y_i} \cdot \frac{100}{n},$$

where $Y_i$ and $F_i$ represent actual price and forecast price on day $i$, and $n$ is the total number of forecasting days. Lawrence et al. [15] determined the judgment scale of forecasting accuracy by using MAPE as demonstrated in Table 2.

Table 2. The scale of judgment of forecast accuracy using MAPE

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highly accurate</td>
<td>MAPE &lt; 10%</td>
</tr>
<tr>
<td>Good accurate</td>
<td>10% ≤ MAPE &lt; 20%</td>
</tr>
<tr>
<td>Reasonable</td>
<td>20% ≤ MAPE &lt; 50%</td>
</tr>
<tr>
<td>Inaccurate</td>
<td>MAPE ≥ 50%</td>
</tr>
</tbody>
</table>
3. Description of Data

In this work, the market indices of Saudi Arabia that is called *Tadawul All Share Index (TASI)* were selected. TASI reflects the performance of economic growth and financial stability of Saudi Arabia.

The data is available online at http://www.investing.com. The daily open prices from 1st January 2017 to 31st December 2017 are studied with total observations of 250 days. This duration is selected due to the appearance of long memory, i.e., $H > 0.5$. Return series (in logarithm) is considered to deal with high volatility in the data. Figures 1 and 2 show the open prices and its return series.

![Figure 1. Daily open price series of TASI from 1st January 2017 to 31st December 2017.](image)

![Figure 2. Return series of TASI from 1st January 2017 to 31st December 2017.](image)
4. Forecasting TASI

In the beginning of this section, we illustrate the referring of each of the following parameters $H_1$, $H_2$, $\mu$, $\beta$, $m$ and $\alpha$, which represent Hurst index of GFBM, Hurst index of FOU process, mean of return, volatility of volatility, mean of volatility and mean reverting parameter, respectively.

In this work, we forecast daily index prices of TASI for year 2018. Depending on the daily index prices in 2017, we obtained the values of initial parameters as follows: $H_1 = 0.57$, $H_2 = 0.63$, $\mu = 0.000011$, $\beta = 0.00019$, $m = 0.000055$ and $\alpha = 2.45$.

First, we compute the value of volatility using three different formulas that are listed in Table 1, as presented in Table 3.

**Table 3.** Volatilities according to the formulas of simple ($S$), log ($L$) and stochastic (STO)

<table>
<thead>
<tr>
<th>Volatility type</th>
<th>$S$</th>
<th>$L$</th>
<th>STO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.00743</td>
<td>0.003221</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

Second, we forecast the open prices by using both GBM and GFBM models as its underlying process via the volatility values listed in Table 3. The forecasted prices were computed by using six models include GBM-S, GFBM-S, GBM-L, GFBM-L, GBM-STO and GFBM-STO.

Third, MAPE values are compared where the smallest value is considered the best. Table 4 represents the level of accuracy of the models.

**Table 4.** The level of accuracy ranking for forecasting model of S&P 500

<table>
<thead>
<tr>
<th>Rank</th>
<th>Model</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GBM-L</td>
<td>7.7099%</td>
</tr>
<tr>
<td>2</td>
<td>GFBM-S</td>
<td>7.8702%</td>
</tr>
<tr>
<td>3</td>
<td>GFBM-L</td>
<td>7.8703%</td>
</tr>
<tr>
<td>4</td>
<td>GBM-S</td>
<td>7.8898%</td>
</tr>
<tr>
<td>5</td>
<td>GFBM-STO</td>
<td>7.9501%</td>
</tr>
<tr>
<td>6</td>
<td>GBM-STO</td>
<td>7.9578%</td>
</tr>
</tbody>
</table>
5. Results

Table 4 showed that all values of MAPE are relatively close, indicating that forecasts by both GBM and GFBM models are highly accurate (less than 10%). However, we can observe that GFBM models are more accurate than GBM models when the volatility is computed by STO and $S$. These findings are consistent with Painter [19], Willinger et al. [27], Grau-Carles [10] and Rejichi and Aloui [20] according to which long memory models are best suited in empirical analysis. Furthermore, the model GBM-L performs most accurate, whereas GBM-STO performs the worst. By referring to Tables 3 and 4, the readers can observe an inverse relationship between the value of volatility and the level of accuracy.

Figures 3-8 compare the return of actual prices versus forecasted return prices obtained by GBM and GFBM models with three different volatilities as previously mentioned in Table 1. These figures indicate that the forecasted prices are closer together and less fluctuated than the actual prices. Moreover, the forecasted prices that obtained depending on stochastic volatility are more stable than the forecasted prices that obtained depending on simple volatility and log volatility.

![Actual Return vs GBM-S Return](image)

**Figure 3.** Actual return vs GBM-S return.
Figure 4. Actual return vs GFBM-S return.

Figure 5. Actual return vs GBM-L return.

Figure 6. Actual return vs GFBM-L return.
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References


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