Clique Domination in an Undirected Graph $G_{m,n}$

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ABSTRACT

In this paper we discuss about the undirected graph $G_{m,n}$ whose set of vertices is given as $V = I_n = \{1,2,3,\ldots\}$ where $u,v \in V$ are adjacent if and only if $u \neq v$ and $u + v$ is not divisible by $m$ where $m$ belongs to Natural numbers greater than 1. We discuss in this paper about clique domination of some special cases of $G_{m,n}$ for $m$ and $n$.

Keywords: dominating set, domination number, clique dominating set, clique domination number.

1. INTRODUCTION

Graph theory is considered as one of the most flourishing branch both in modern mathematics as well as computer applications. Recently Theory of domination became an area which attracted many researchers due to its wide scope. Historically the domination type problem originated from chess. The historical roots of domination in graph theory dates back to 1862, when the chess master C.F. de Jaenisch wrote a treatise\(^4\), in which he suggested the number of queens required to attack every square on a $n \times n$ chess board. The concept of domination number was given by Berge in 1958. Ore gave the name for the same concept as “dominating” set in 1962. The book\(^3\) lists over 1200 papers on domination in graphs. More than 75 variations on dominations were cited in\(^1\).

Definition 1.1: An undirected graph $G_{m,n}$ is a graph whose set of vertices is given as $V = I_n = \{1,2,3,\ldots\}$ where $u,v \in V$ are adjacent if and only if $u \neq v$ and $u + v$ is not divisible by $m$ where $m$ belongs to Natural numbers greater than 1.
Definition 1.2: A set $D$ of vertices in a simple, undirected graph $G_{m,n} = (V,E)$ is a dominating set of $G_{m,n} = (V,E)$ if every vertex in $V-D$ is adjacent to some vertex in $D$. The domination number of a graph $G$ is the minimum number of vertices in a dominating set. The minimum cardinality among the dominating sets of $G$ is called the domination number of $G$ and denoted by $\gamma(G)$.

Definition 1.3: A complete sub graph or a clique is an induced sub graph such that there is an edge between each pair of vertices in the subgraph.

Dominating sets have applications in a variety of fields, including communication theory and political science.

2. SOME BASIC PROPERTIES OF UNDIRECTED GRAPH $G_{m,n}$:

An Undirected graph $G_{m,n}$ was first introduced and studied by Dr. Ivy Chakrabarty, some of the properties of $G_{m,n}$ are:

2.1. The graph $G_{m,n}$ is connected where $m,n \in N$ and $m,n > 1$.

2.2. $G_{m,n} \cong K_3$ if and only if $n=3$ and $m \geq 6$.

2.3. If $n=m-1$ where $m$ is odd and $k=n/2$ then $G_{m,n}$ is a complete $k$-partite graph.

2.4. The graph $G_{m,n}$ is Eulerian where $m$ is odd and $n=m-1$.

2.5. The graph $G_{m,n}$ is complete when $m \geq 2n$.

2.6. The graph $G_{m,n}$ is Hamiltonian when $m \geq 2n$ with $n \geq 3$.

3. CLIQUE DOMINATION

Let $G$ be a nontrivial connected graph. A dominating set $D$ of $V$ is a clique dominating set of $G_{m,n}$ if the induced subgraph $\langle D \rangle$ of $D$ is complete. The minimum cardinality of a clique dominating set of $G$, denoted by $\gamma_{cl}(G)$, is called the clique domination number of $G$. A clique dominating set of $G$ with cardinality $\gamma_{cl}(G)$ is called a clique set of $G$. The concept of clique domination was first studied by Cozzens and Kelleher in. Domination and other variations of domination can be found in and .

Theorem 3.1: Let $G_{m,n}$ be a connected graph, then $\gamma_{cl}(G_{m,n}) = 1$ if and only if $\gamma(G_{m,n}) = 1$.

Theorem 3.2: Let $m,n \in N$ and $m,n > 1$ then the graph $G_{m,n}$ is Clique dominating graph if $m=n$ and $\gamma_{cl}(G_{m,n}) = 2$.

Proof: Let $G_{m,n}$ be a graph with $m=n$ and $m,n > 1$. Let $D$ be a dominating set of $G_{m,n}$ with minimum cardinality 2. Now we have to prove that $D$ is a CDS with $\gamma_{cl}(G_{m,n}) = 2$.

We know that any set with two vertices with an edge is complete graph. So when $D$ has minimum cardinality 2 then the induced graph $\langle D \rangle$ is also complete. So by definition $D$ becomes CDS. Hence $\gamma_{cl}(G_{m,n}) = 2$. 

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For the graph $G_{6,6}$, $\gamma_{cl}(G_{6,6}) = 2$.

**Theorem 3.3**: Let $m=2n$ then the graph $G_{m,n}$ has CDS with $1 \leq \gamma_{cl}(G_{m,n}) \leq n - 1$.

**Proof**: If $m=2n$ then $G_{m,n}$ is a complete by 2.5.
Let $D$ be a dominating set of $G_{m,n}$.
Since $G_{m,n}$ is a complete graph then the induced graph $\langle D \rangle$ is also complete.
By definition, $\langle D \rangle$ will become CDS.
Let $D$ has one vertex.
We know that a graph with one vertex is complete. So cardinality of CDS is 1.
Now let $D$ has 2 vertices and an edge. Then $D$ becomes complete and hence $D$ is a CDS with cardinality 2.
So in general when $D$ is a complete graph with $n$ vertices it has cardinality $n-1$.
Hence $1 \leq \gamma_{cl}(G_{m,n}) \leq n - 1$.

For the graph $G_{10,5}$, $1 \leq \gamma_{cl}(G_{m,n}) \leq n - 1$

**Theorem 3.4**: Hamiltonian graph $G_{m,n}$ for $m \geq 2n$ and $n \geq 3$ has a CDS.

**Proof**: From Theorem 3.2 any graph $G_{m,n}$ with $m=2n$ is a complete graph.
Also complete graph has a CDS. Hence when $m \geq 2n$ and $n \geq 3$, $G_{m,n}$ has a CDS.

**Theorem 3.5**: For any graph $G_{m,n}$ where $m$ is odd and $n=m-1$, then $G_{m,n}$ has no CDS.

**Proof**: From 2.3 above, $G_{m,n}$ is a $k$-partite graph with $k = \frac{n}{2}$ and every partite set is a dominating set.
Also this graph has $P_5$ or $C_5$ then by Theorem 2 in [1] $G_{m,n}$ has no dominating clique.

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CONCLUSION

In this paper we have proved that the clique domination number varies for different values of $m$ and $n$ of graph $G_{m,n}$.

REFERENCES